Effect of System States and Parametric Sensitivity Analyses on Power System Modal Interactions

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Abstract

In this paper, the parametric sensitivities of a structure preserving a power system model are derived in the interest of effective model reduction. This parametric sensitivity analysis defines the sensitivity of a steady state point against the variation of a particular system parameter. The parametric sensitivities are derived here by studying the effects changes in a given parameter over the system natural modes of the variations on the small signal stability and the results obtained were confirmed. Also the correlations between different inherent modes resulting from small perturbation stability have been confirmed.

Keywords: Participation Factor, Sensitivity analysis, modal interaction, power system stability, small perturbation analysis.

1. Introduction

The state of stability of power system is not only defined by the rate of decaying oscillations or its positive damping but also by the stability of its parameters. It is also important to identify the state variable and parameters that contribute to developing a particular type of the effect of instability. In this work a comprehensive dynamic model is developed to study the system parameter over the resulting system stability using a dq0 model. The electromechanical oscillations and their damping, as well as dynamic voltage stability between a remotely located synchronous machine and the remaining of a power system are studied. A single-machine infinite-bus system case that investigates only local oscillations' sensitivities to parametric variation is introduced. The effect of parameters variation in machine parameters such as R_a, R_f, R_{kd}, R_{kq}, H, tie line parameters such as R_e, X_e and loading conditions such as P, Q, over power system natural modes is analyzed.

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Due to the possible large number of modal interactions it is often necessary to construct a reduced order model for dynamic stability studies by retaining only modes of interest while preserving the consistency of the analysis. The appropriate identification of the state variables significantly participating in a given mode becomes effective in defining the end form of the reduced order model. This requires a tool for identifying the state variables that have a significant participation in a selected mode. It is natural to suggest that the significant state variables for an eigenvalue λ_i are those that correspond to large entries in the corresponding eigenvector υ_i. Verhese et al. [1] have suggested a related measure of a state variable participation factors *(PF)*. Participation factor analysis assists in the identification of how much each dynamic state variable affects a given mode or eigenvalue.

2. Preliminary

In power system analysis, mathematical equations that represent the power system under study in the dq0 model are generally described by seven nonlinear differential equation for the case where only one damper circuit on each axis is considered as shown in appendix [I]. This set of nonlinear differential equations can be linearized around a quiescent operating point on a basis of small perturbation. The mathematical description thus obtain yield a linearized system of differential equation with constant coefficient which will take the form:

$$\dot{X} = AX + BU$$

The Participation factor is a sensitivity measure of an eigenvalue to a diagonal entry of the linearized system matrix, and can be defined as

$$PF_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}}$$

Where, PF_{in} is the participation factor relating the k^{th} state variable with the i^{th} eigenvalue, a_{kk} is the k^{th} entry of the system matrix A.

It is well known that if A has all its eigenvalues λ_i $(i=1,2,\ldots\ldots,m)$, then it will have m corresponding linearly independent $m\times 1$ eigenvectors υ_i $(i=1,2,\ldots\ldots,m)$ satisfying the relation

$$A\upsilon_i = \lambda_i \upsilon_i, (i = 1,2,\ldots\ldots, m)$$

Here υ_i is called the right eigenvector associated with λ_i. There also exists a vector ω_i^t satisfying the relation,

$$\omega_i{}^t A = \omega_i{}^t \lambda_i, (i = 1,2,.......m)$$

Where t denotes matrix transposition and this vector is the left eigenvector.

Hence *PF* can be defined as

$$PF_{ki} = \frac{\omega_{ki}\upsilon_{ki}}{\omega_i{}^t \upsilon_i} \quad (i = 1,2,.......m)$$

Where ω_{ki} and υ_{ki} are k^{th} entries in the right and left eigenvectors associated with the i^{th} eigenvalue λ_i. Equivalence between two definitions of the participation factor can be derived considering the system

$$[A - \lambda_i I]\upsilon_i = 0.0$$
$$\omega_i{}^t [A - \lambda_i I] = 0.0$$

To examine the sensitivity of the eigenvalue λ_i of the diagonal element of the system modal matrix A; The perturbed equations will read;

$$(A + \Delta A)(\upsilon_i + \Delta\upsilon_i) = (\lambda_i + \Delta\lambda_i)(\upsilon_i + \Delta\upsilon_i)$$
$$[A\upsilon_i] + [\Delta A\upsilon_i + \Delta\upsilon_i A] + [\Delta\upsilon_i \Delta A] = [\lambda_i \upsilon_i] + [\Delta\lambda_i \upsilon_i + \Delta\upsilon_i \lambda_i] + [\Delta\upsilon_i \Delta\lambda_i]$$

After appropriate mathematical manipulations, will yield

$$\omega_i{}^t \Delta A \upsilon_i = \omega_i{}^t \Delta\lambda_i \upsilon_i$$

Assuming that the k^{th} diagonal element of matrix *A* is perturbed so that $\Delta A = \Delta a_{kk}$, hence

$$A = \begin{bmatrix} a_{11} & a_{12} & ...... & a_{1m} \\ a_{21} & a_{22} & .... & a_{2m} \\ .. & ... & a_{kk} & .... \\ a_{m1} & ... & ... & a_{mm} \end{bmatrix}$$

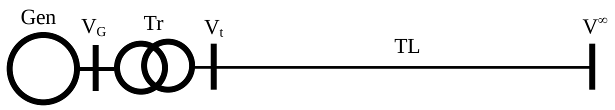
$$\Delta A = \begin{bmatrix} 0 & 0 & ...... & 0 \\ 0 & 0 & .... & 0 \\ .. & ... & \Delta a_{kk} & .... \\ 0 & ... & ... & 0 \end{bmatrix}$$

Now the sensitivity of the eigenvalue λ_i with respect to diagonal elements of the matrix A is related to the PF as follows:

$$\frac{\Delta\lambda_i}{\Delta a_{kk}} = \frac{\omega_{ki}{}^t \upsilon_{ki}}{\omega_i{}^t \upsilon_i} = PF_{ki}$$

3. Study Case

The above mentioned methodology is applied to a single-machine infinite-bus system whose detailed power system model is described in appendix (I).



This formulation includes both the generator electrical and mechanical models. The analysis is performed after the machine is undergoing a small perturbation; the state space representation for system model in small perturbed form is given in appendix (II). The perturbed system equations in matrix form is given as:

$$E\Delta X^{\bullet} = F\Delta X$$

$$\Delta X^{\bullet} = E^{-1}F\Delta X$$

$$= Asys\Delta X$$

Where $Asys = E^{-1}F$ and

E denotes the matrix of coefficient of the derivative of the perturbed state variables

F denotes the matrix of coefficient of the perturbation state variables

As no feedback control action is considered, it goes without saying that the matrix Δu of the perturbed input state variables becomes a null matrix

4.Results

1- Eigen vector P.F

The eigenvalues and eigenvectors of the perturbed system matrix A_{sys} are extracted. The different modes obtained are listed below in table I.

Table (I): Eigenvalues and Eigenvectors (single M/C- IBB).

		Stator Modes	**Rotor Modes**			
			Electrical Modes			**Mechanical Modes**
			Field	**D – Damper**	**Q- damper**	**Hunting**
Eigenvalues (λ_i)		-7.0312±376.95i	-0.2404	-32.057	-9.6926	-0.45522±9.1372i
Normalized Eigenvectors(υ_i)	i_d	**0.5291 - 0.0037i**	**-0.35166**	-0.043367	-0.036292	-0.028037-0.030995i
	i_q	-0.0009 + 0.5259i	0.077895	0.00022283	0.22864	-0.035622-0.022153i
	i_f	**0.2306 - 0.0215i**	**-0.87814**	**0.6559**	-0.040568	-0.020724-0.02995i
	i_{kd}	**0.2792 + 0.0197i**	-0.030526	**-0.75081**	0.0069605	-0.0061565+0.001832i
	i_{kq}	-0.0087 + 0.5063i	-0.0032287	0.00026214	**0.55143**	-0.034851+0.0008586i
	ω	**-0.2323 + 0.0354i**	-0.073243	-0.06472	**0.79617**	**-0.71159+0.68979i**
	δ	0.0001 + 0.0006i	**0.30467**	0.0020189	-0.082142	-0.071436-0.081437i

The participation factor technique is there after applied to the eigen vectors. The results of which are given in table II and are named after the predominant dynamic variable.

Table II - Participation Factor of State Variables in System Mode

State Variables	Stator & Network mode	Rotor Modes			
		Field mode	D-damper	Q-damper	Hunting
i_d	0.34793	0.399150	0.02908	0.11366	0.066903
i_q	0.34561	0.011011	0.00000	0.21351	0.050971
i_f	0.06999	0.481500	0.18479	0.00900	0.062181
i_{kd}	0.08459	0.075657	0.78555	0.00568	0.010270
i_{kq}	0.15172	0.025197	0.00000	0.36963	0.044229
Ω	0.00000	0.007025	0.00027	0.27184	0.382730
Δ	0.00014	0.000462	0.00030	0.01670	0.382720

2- Effect of ParameterVariation

The effect of parameter variation on each mode is applied to the set of modes to work out their sensitivity of each of the predominate. This analysis will be performed in three major steps:

a.Study of the variations in generator transformer parameters.
b.Study of the variations in transmission system parameters.
c.Study of the variations in initial loading condition.

a) **Effect of Variation in Generator/Transformer Parameters**

To assess the effect of the parametric variation on the inherent modes, eigenvalue program is to be executed several times, with a certain step of variations of parameters under consideration. The selected machine parameters under consideration are armature R_a or transformer resistance R_{tr}, field resistance R_f, direct and quadrature axis damper resistance R_{kd}, R_{kq} respectively, rotor inertia (2H). The system inherent modes are represented by recording the variation in both damping factors, and the angular frequency of oscillations. The most sensitive modes are to be shown.

b) **Effect of Variation in Loading Condition**

a. Effect of Variation in Circuit External impedance

Here the eigenvalue program is to be repeated several times, keeping the constant variation in both external resistance and reactance of the tie line. The most sensitive modes are to be shown.

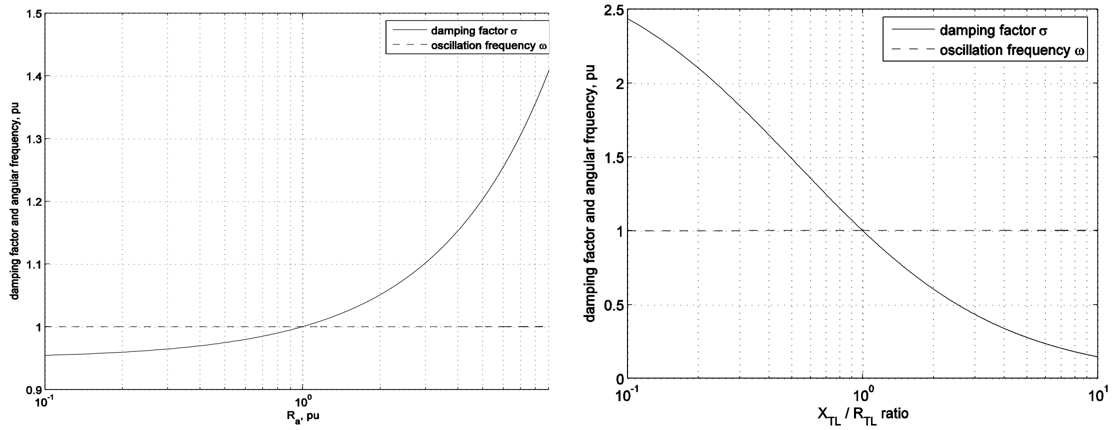


Fig. 2- Effect of variation in R_a and X_{TL}/R_{TL} ratio on the ***stator*** **mode.**

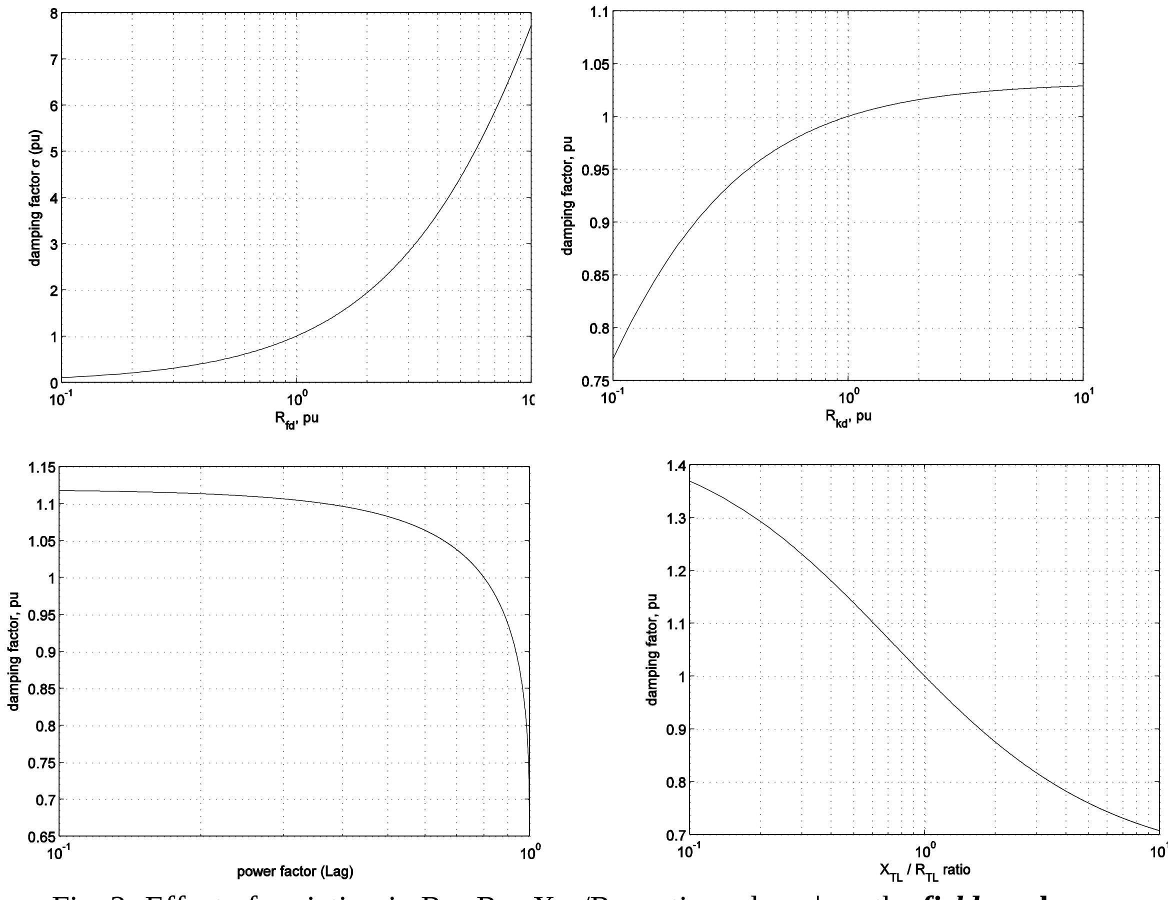


Fig. 3- Effect of variation in R_{fd}, R_{kd}, X_{TL}/R_{TL} ratio and cosϕ on the ***field*** **mode.**

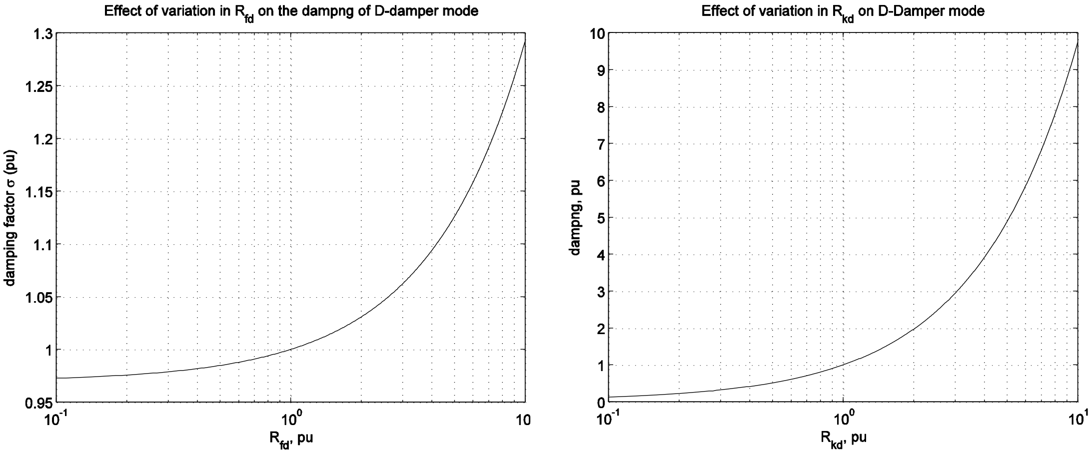


Fig. 4- Effect of the variation in R_{fd} and R_{kd} on the **D-damper mode.**

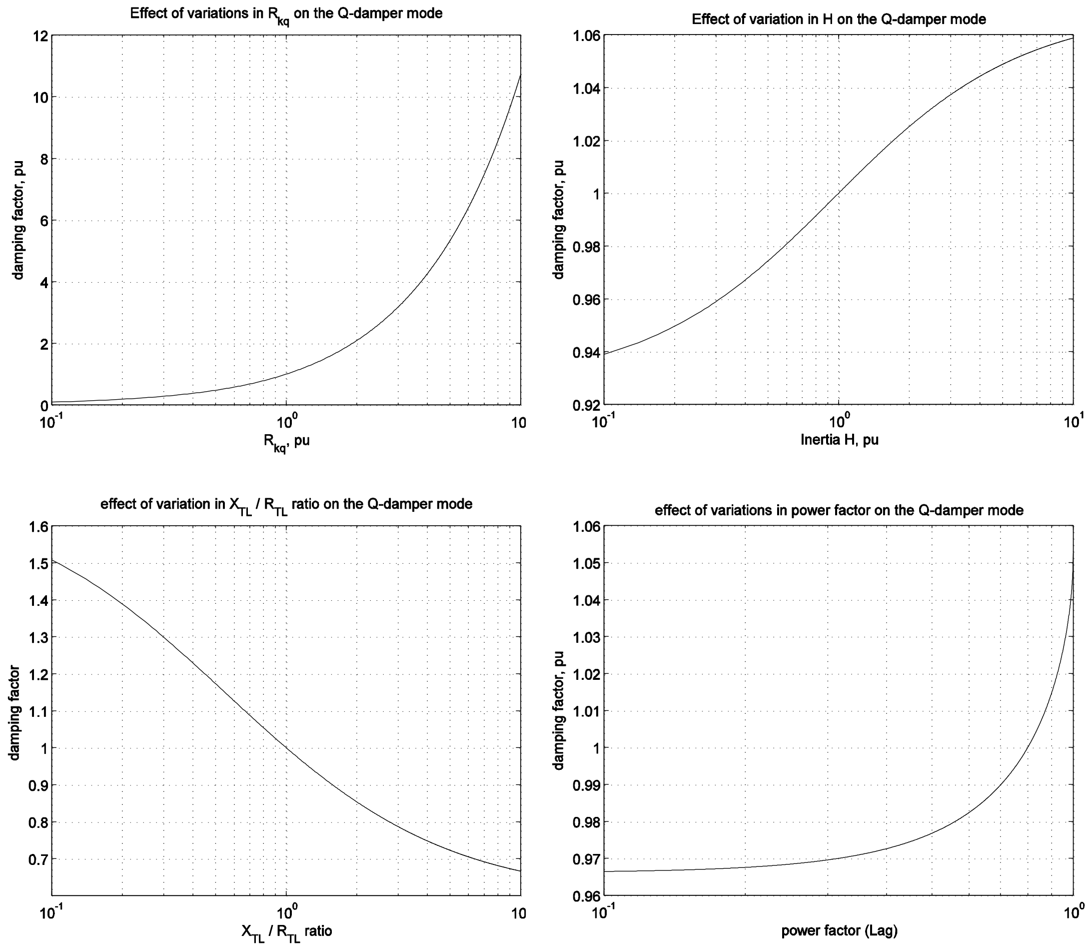


Fig. 5- Effect of the variation in R_{fd},H, X_{TL}/R_{TL} ratio and cosϕ on the **Q-damper mode.**

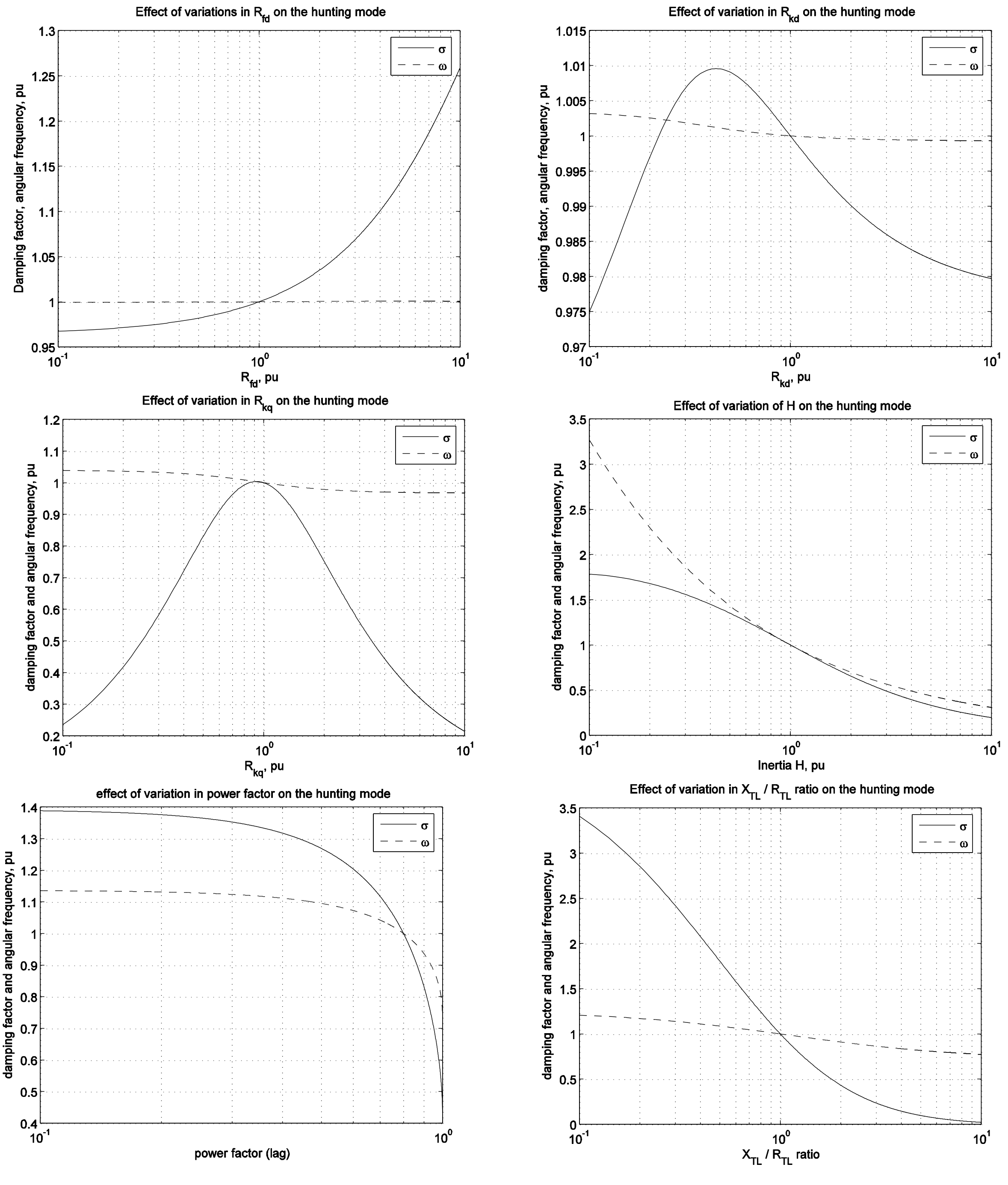


Fig. 6- Effect of the variation in R_{fd},R_{kd},R_{kq},H, X_{TL}/R_{TL} ratio and cosϕ on the **Hunting mode.**

5. Conclusion

6. References

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Appendix I

System Data from reference [21] on a basis of 500 KV, 10 GW

Generator Data: $X_d = 1.51$, $X_{ad} = 1.31$, $X_l = 0.2$, $X_q = 1.49$, $X_{aq} = 1.29$, $X_f = 1.42$, $X_{kd} = 1.4$, $X_{kq} = 1.34$, $R_a = 0.0015$, $R_f = 0.00063$, $R_{kd} = 0.0153$, $R_{kq} = 0.0207$, $2H = 5.24\sec$, $D = 0.0$

Transformer Data: $R_{tr} = 0.003 - 0.0045$, $X_{tr} = 0.135$

Transmission Line Data $R_{tl} = 0.02 - 0.027$, $X_{tl} = 0.905$

Receiving System $R_s = 0.005$, $X_s = 0.3$

Initial loading condition at infinite bus bar, $V^{\infty} = 1.0$, $P_{\infty} = 0.8$, $Q_{\infty} = 0.6$, $\omega_o = 377.0$ rad/sec $P_{\infty} = 0.81$

All values mentioned above are taken in p.u on a basis of machine rating unless otherwise stated

Appendix II

Mathematical Model for a Single Machine –Infinite Bus Bar System

The mathematical description of the transient model for single machine infinite bus bar system shown in fig.1 is given below with state space vector $X^t = [i_d, i_q, i_f, i_{kd}, i_{kq}, \omega_r, \partial]$:

$$\frac{P}{\omega_o}(-X_{dt}i_d + X_{ad}i_f + X_{ad}i_{kd}) = R_t i_d + \frac{\omega_r}{\omega_o}(-X_{qt}i_q + X_{aq}i_{kq}) + V^{\infty}\sin\partial$$

$$\frac{P}{\omega_o}(-X_{qt}i_q + X_{aq}i_{kq}) = R_t i_q - \frac{\omega_r}{\omega_o}(-X_{dt}i_d + X_{ad}i_f + X_{ad}i_{kd}) + V^{\infty}\cos\partial$$

$$\frac{P}{\omega_o}(-X_{ad}i_d + X_f i_f + X_{ad}i_{kd}) = -R_f i_f + V_f$$

$$\frac{P}{\omega_o}(-X_{aq}i_q + X_{kq}i_{kq}) = -R_{kq}i_{kq}$$

$$\frac{2H}{\omega_o}P\omega_r = T_m - T_{em} - \frac{D}{\omega_o}\omega_r, \quad \text{where } T_{em} = \psi_d i_q - \psi_q i_d$$

$$P\partial = \omega_r$$

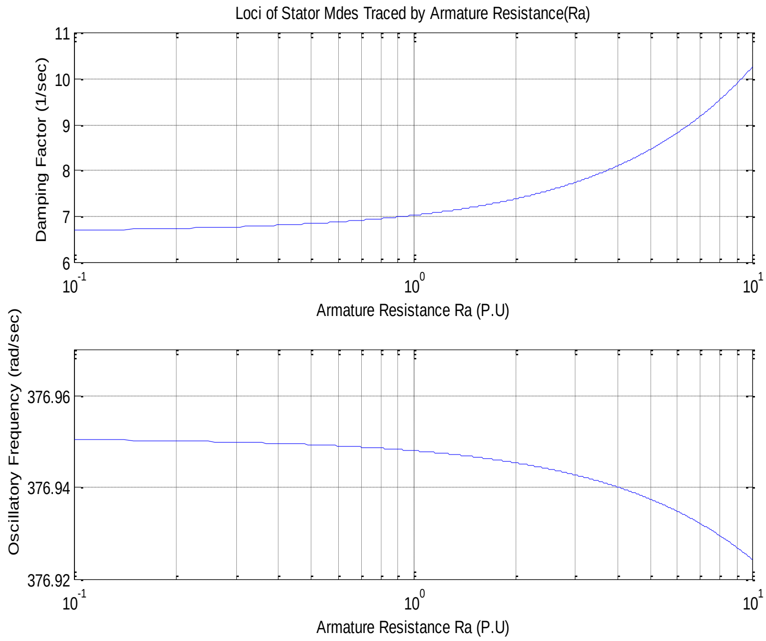
Where $X_{dt} = X_d + X_{tr} + X_{tl}$, $X_{qt} = X_q + X_{tr} + X_{tl}$, $R_t = R_a + R_{tr} + R_{tl}$, $X = X_d - X_q$,

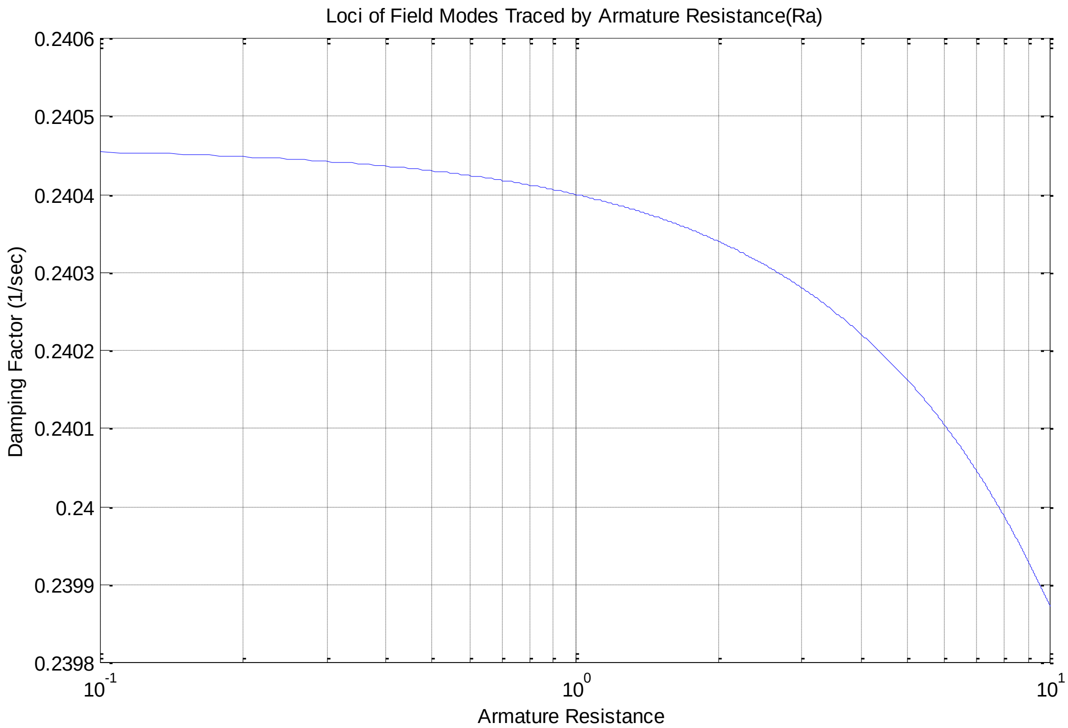
$$T_{em} = -Xi_d i_q + X_{ad}i_f i_q + X_{ad}i_{kd}i_q - X_{aq}i_{kq}i_d$$

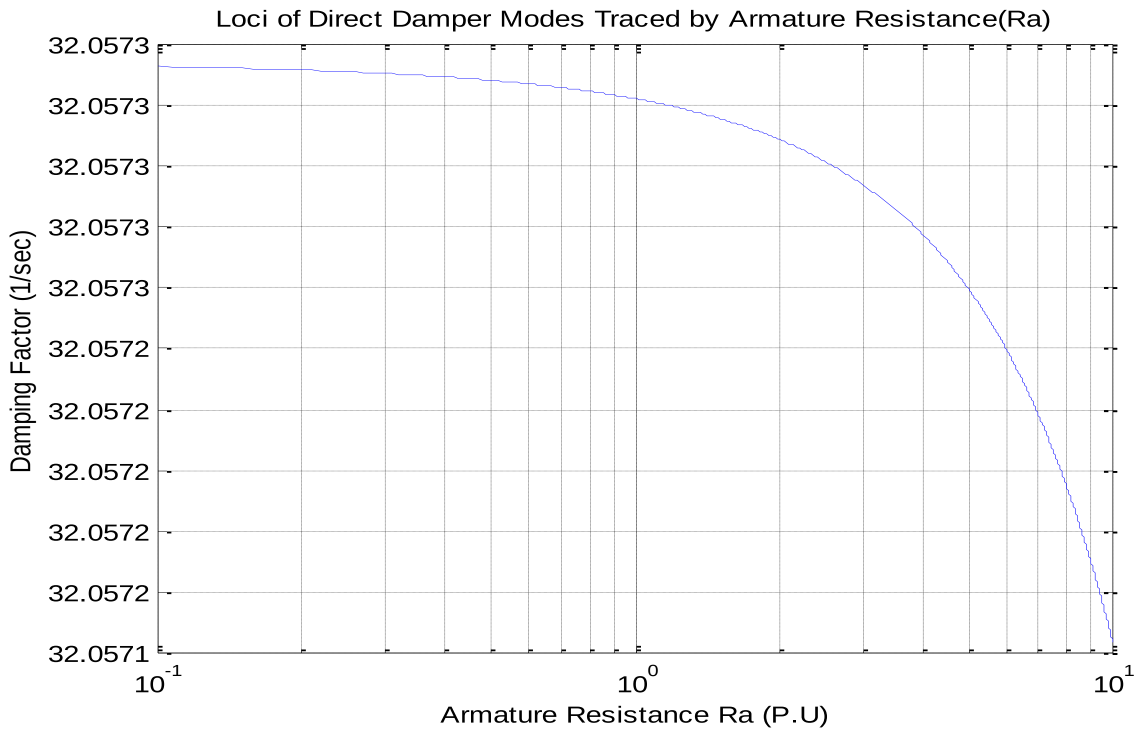
The steady state machine equation is derived from the system of equations given above by making the following substitutions:

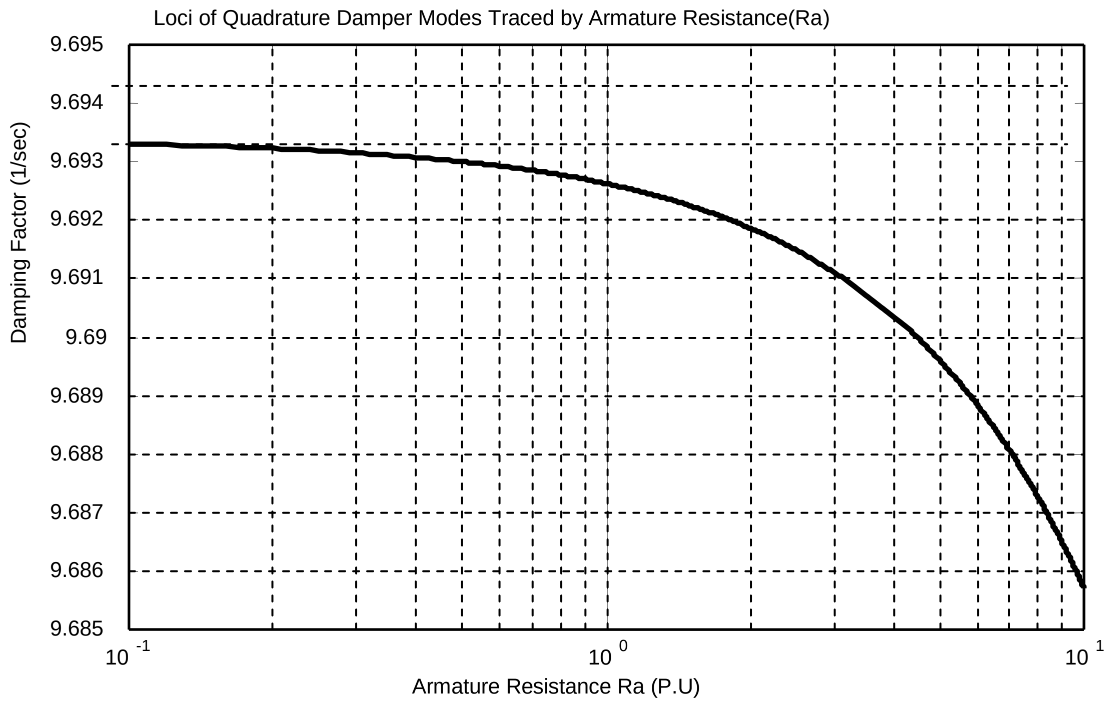
(a)The operator $P = \frac{d}{dt} = 0$, (b)The per unit slip ratio $\frac{\omega_r}{\omega_o} = 1$ and (c) the steady state damper current $i_{kdo} = i_{kqo} = 0$.

$V_{Gd} = X_q I_q - R_a I_q$ and $E_f = V_{Gq} + R_a i_d + X_d I_d = \frac{X_{ad}}{R_f}V_f$

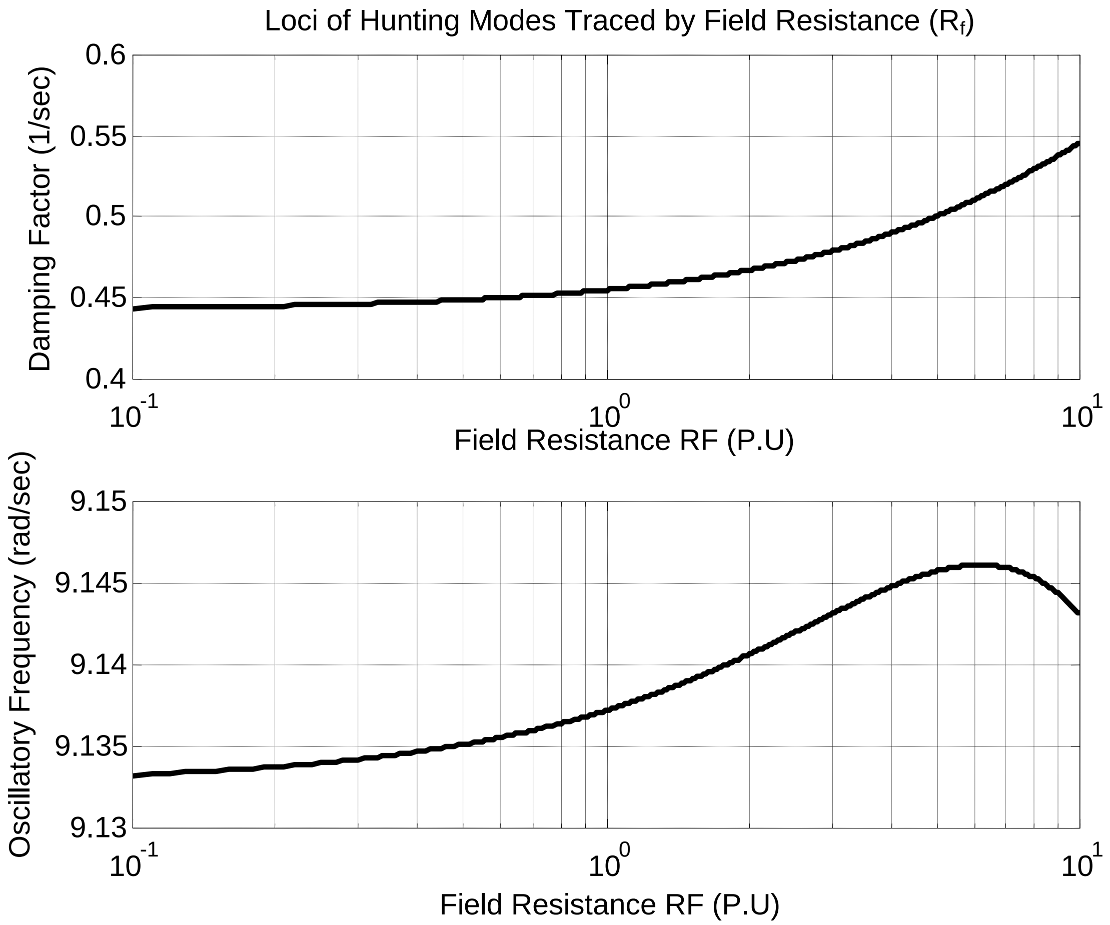

Loci of Stator Mdes Traced by Armature Resistance(Ra)
Damping Factor (1/sec)
Armature Resistance Ra (P.U)
Oscillatory Frequency (rad/sec)
Armature Resistance Ra (P.U)

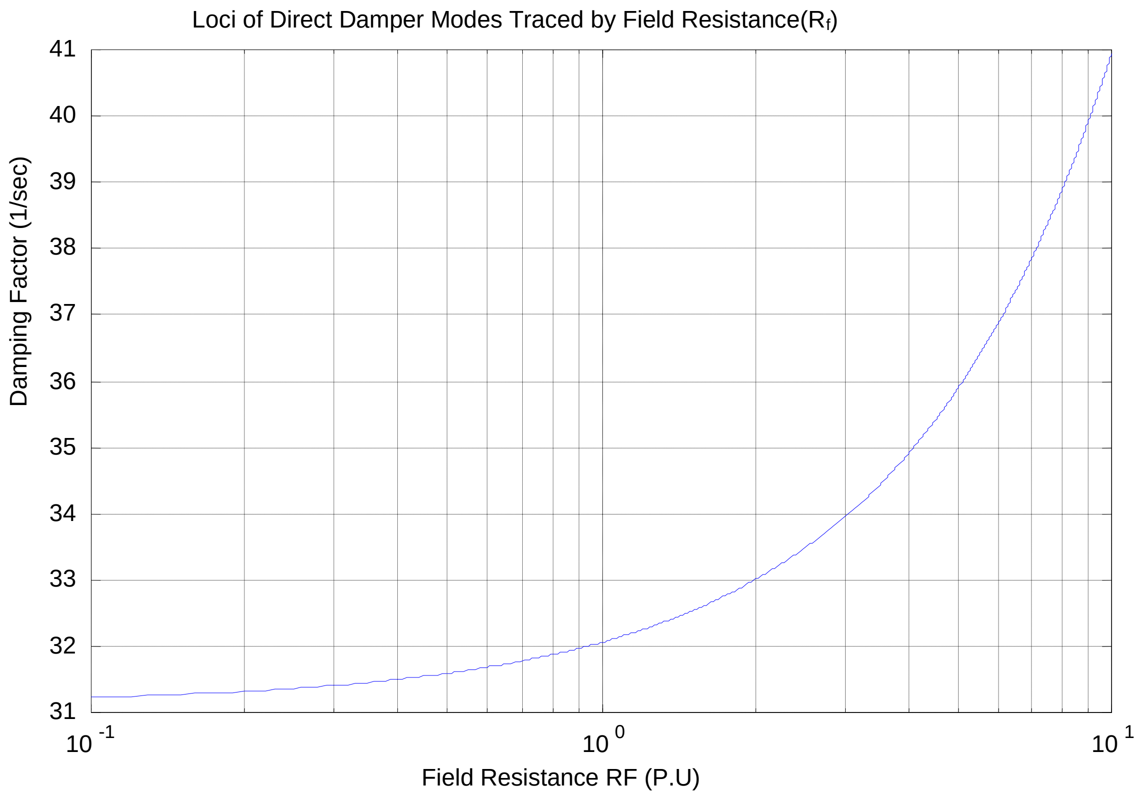

Loci of Field Modes Traced by Armature Resistance(Ra)
Damping Factor (1/sec)
Armature Resistance

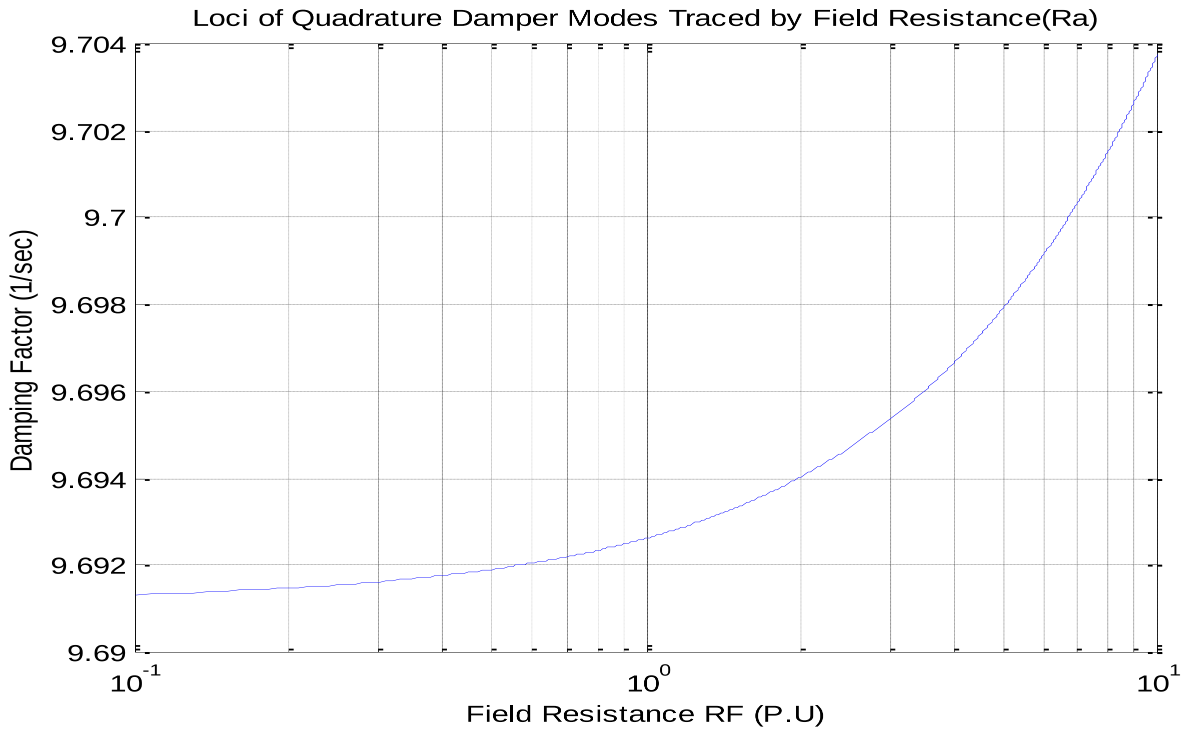

Loci of Direct Damper Modes Traced by Armature Resistance(Ra)
Damping Factor (1/sec)
32.0573
32.0572
32.0571
Armature Resistance Ra (P.U)

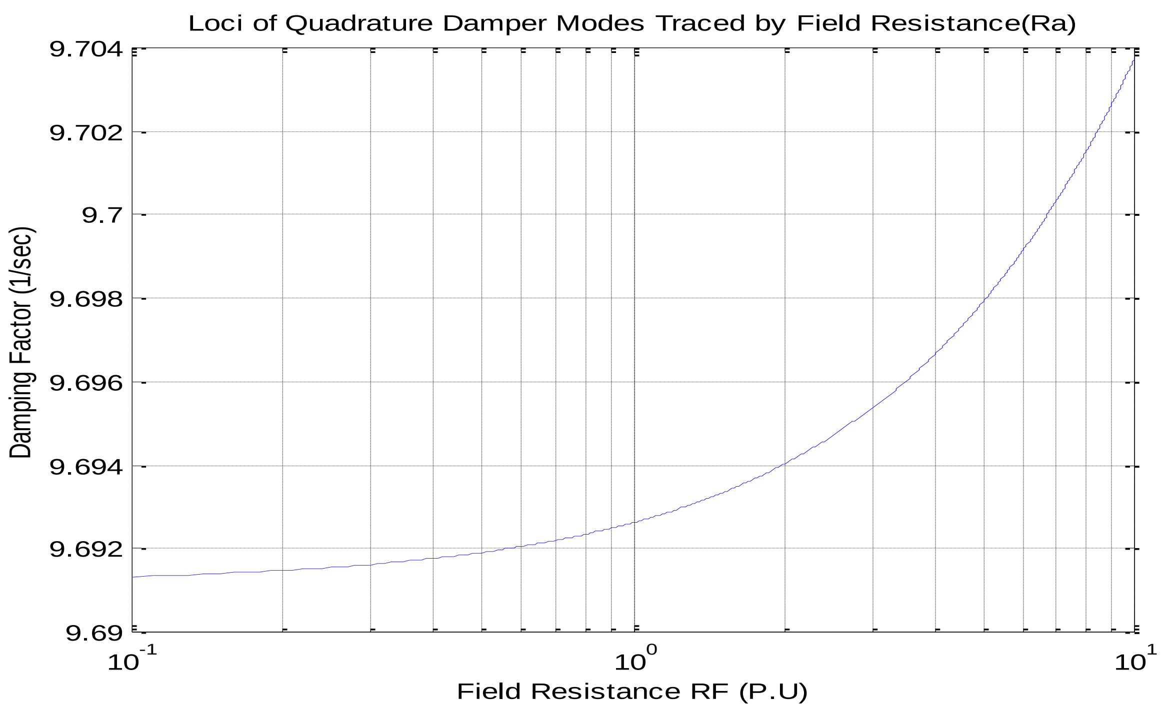

Loci of Quadrature Damper Modes Traced by Armature Resistance(Ra)
Damping Factor (1/sec)
9.695
9.694
9.693
9.692
9.691
9.69
9.689
9.688
9.687
9.686
9.685
Armature Resistance Ra (P.U)

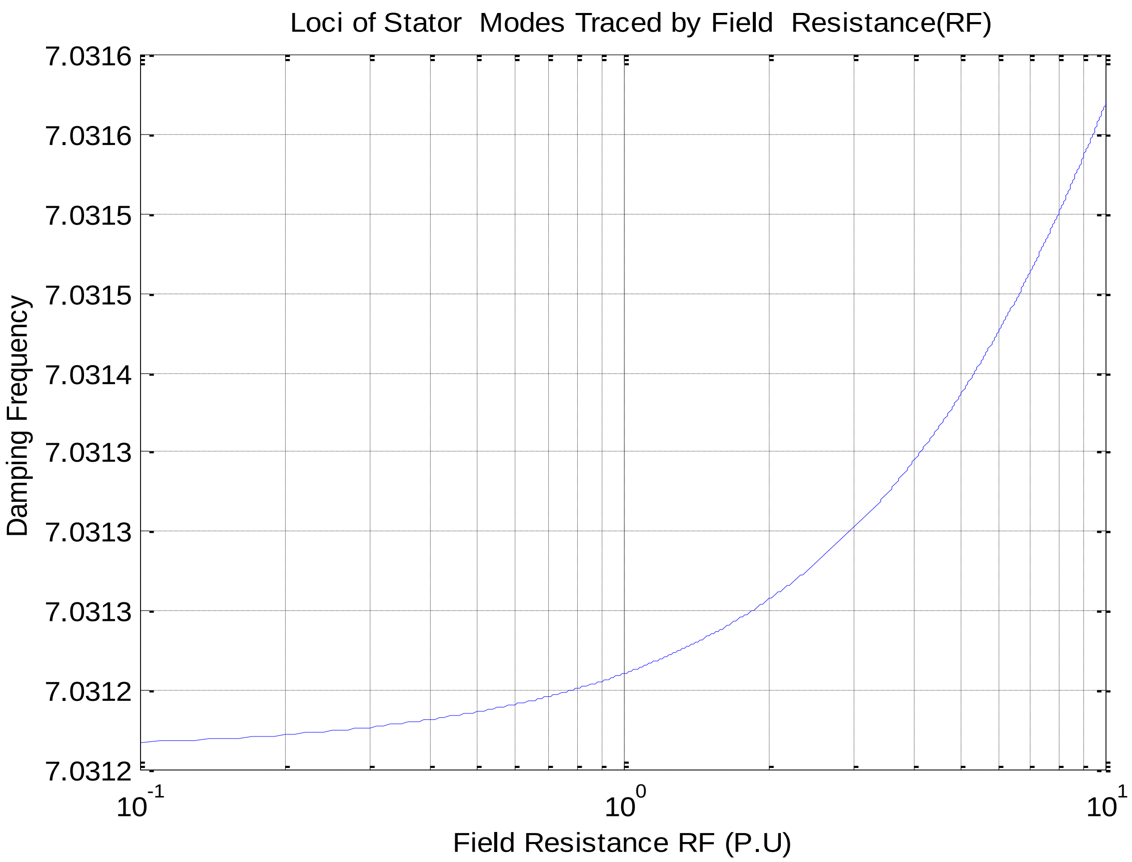
R_f :

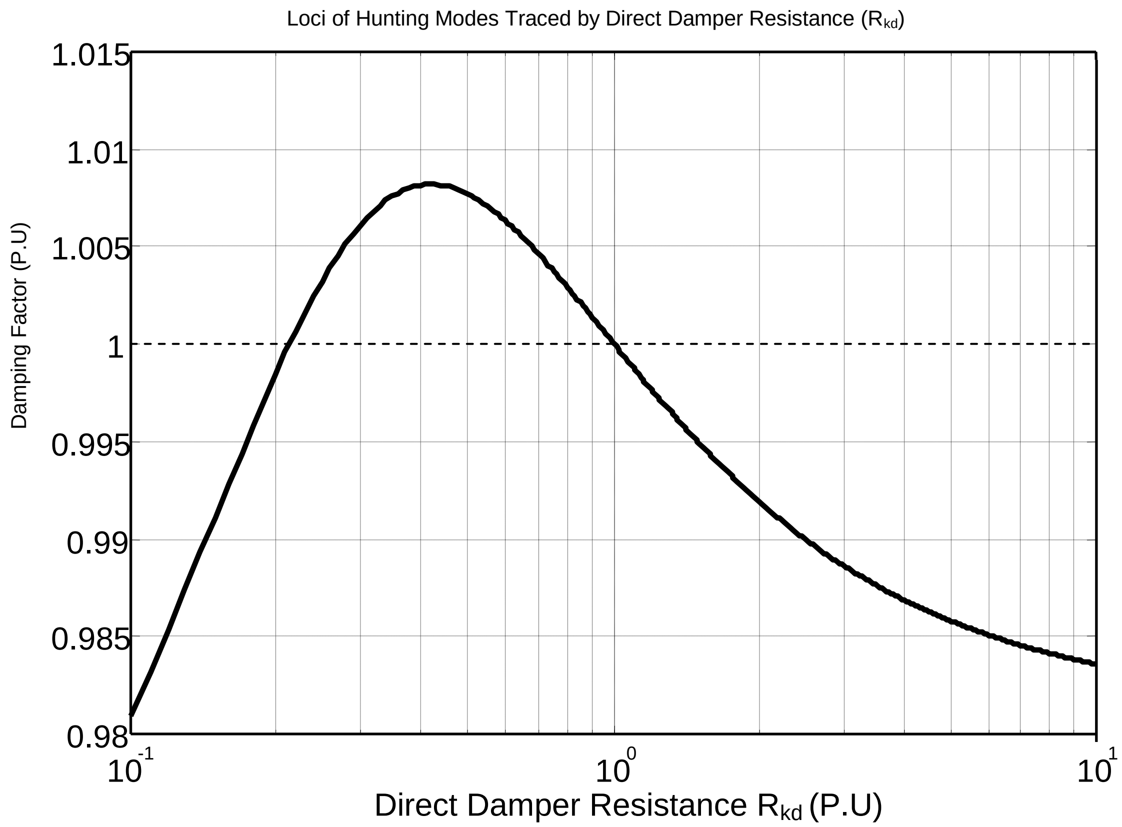

Loci of Hunting Modes Traced by Field Resistance (R_f)
Damping Factor (1/sec)
0.6
0.55
0.5
0.45
0.4
10^{-1}
10^{0}
10^{1}
Field Resistance RF (P.U)
Oscillatory Frequency (rad/sec)
9.15
9.145
9.14
9.135
9.13
10^{-1}
10^{0}
10^{1}
Field Resistance RF (P.U)

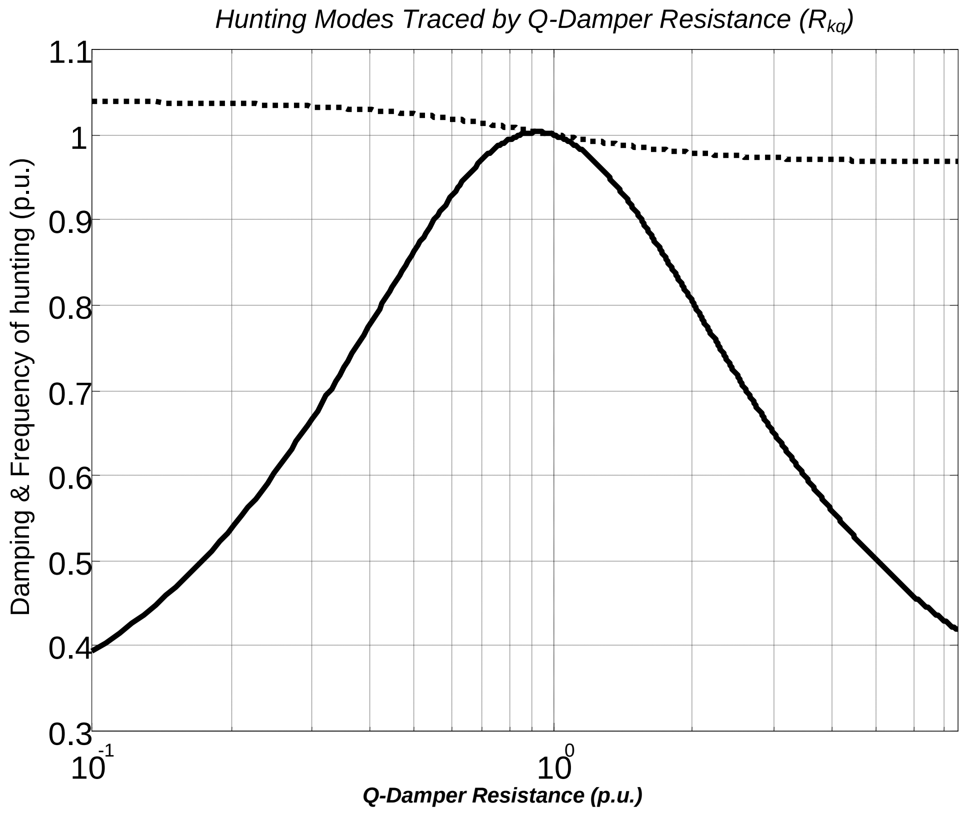

Loci of Direct Damper Modes Traced by Field Resistance(R_f)
Damping Factor (1/sec)
41
40
39
38
37
36
35
34
33
32
31
10^{-1}
10^{0}
10^{1}
Field Resistance RF (P.U)

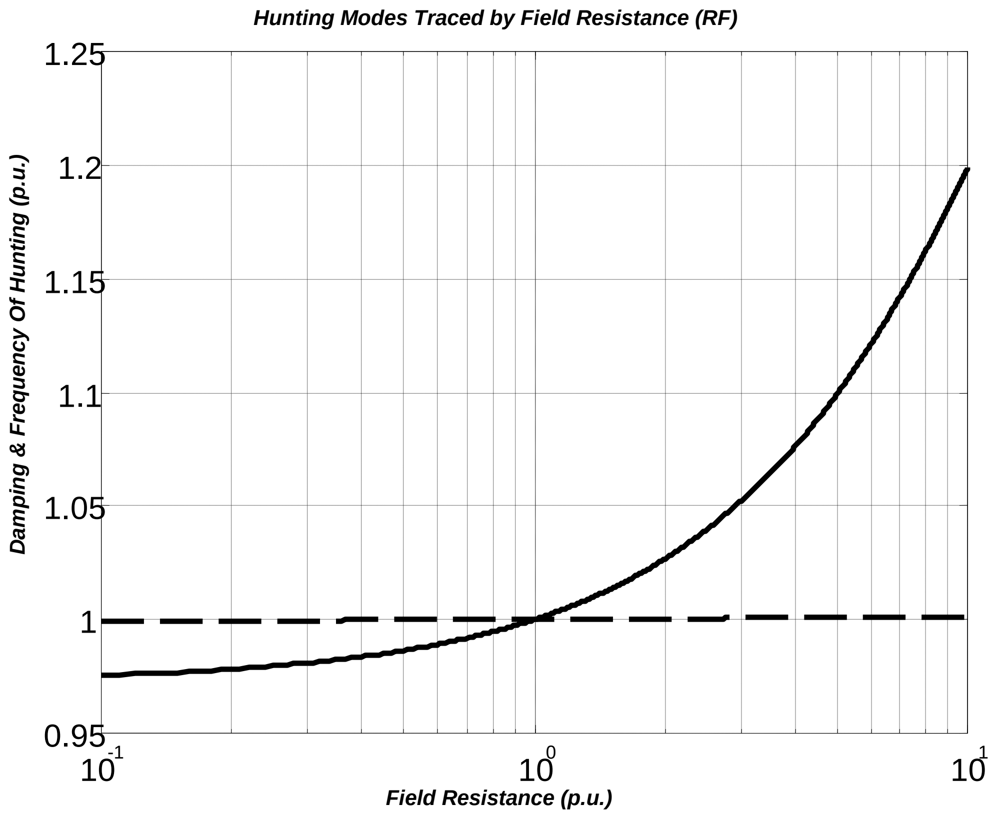

Loci of Quadrature Damper Modes Traced by Field Resistance(Ra)
9.704
9.702
9.7
9.698
9.696
9.694
9.692
9.69
Damping Factor (1/sec)
10^-1
10^0
10^1
Field Resistance RF (P.U)

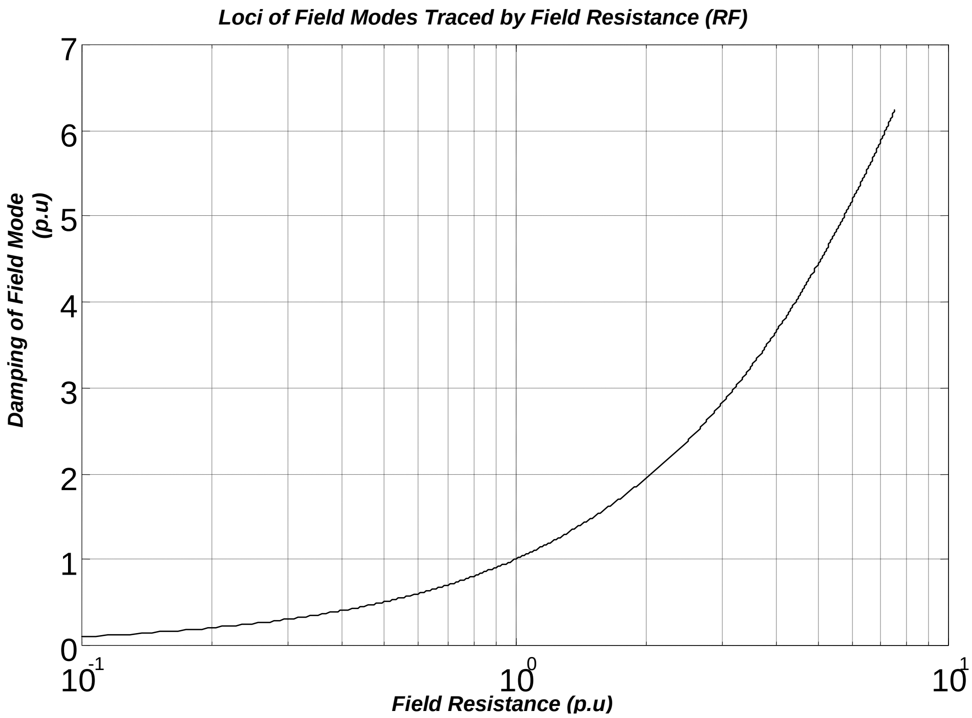

Loci of Quadrature Damper Modes Traced by Field Resistance(Ra)
9.704
9.702
9.7
9.698
9.696
9.694
9.692
9.69
Damping Factor (1/sec)
10^-1
10^0
10^1
Field Resistance RF (P.U)

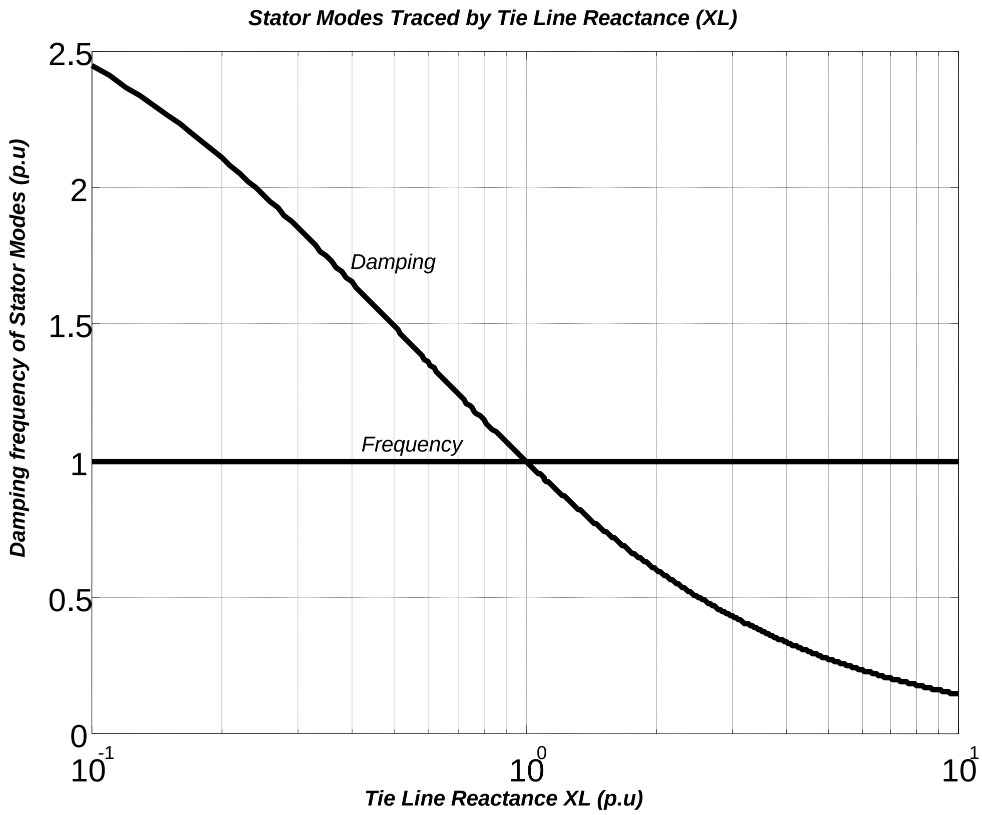

Loci of Stator Modes Traced by Field Resistance(RF)
7.0316
7.0316
7.0315
7.0315
7.0314
7.0313
7.0313
7.0313
7.0312
7.0312
Damping Frequency
10^{-1}
10^{0}
10^{1}
Field Resistance RF (P.U)

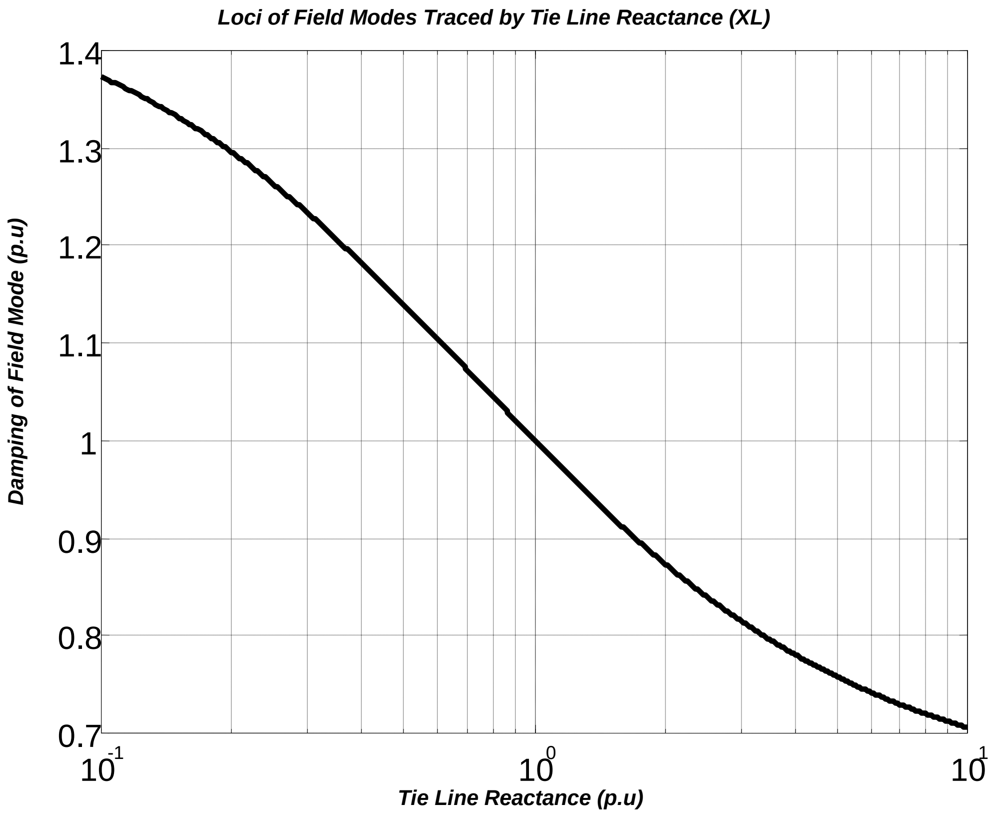

Loci of Hunting Modes Traced by Direct Damper Resistance (R_{kd})
1.015
1.01
1.005
1
0.995
0.99
0.985
0.98
Damping Factor (P.U)
10^{-1}
10^{0}
10^{1}
Direct Damper Resistance R_{kd} (P.U)

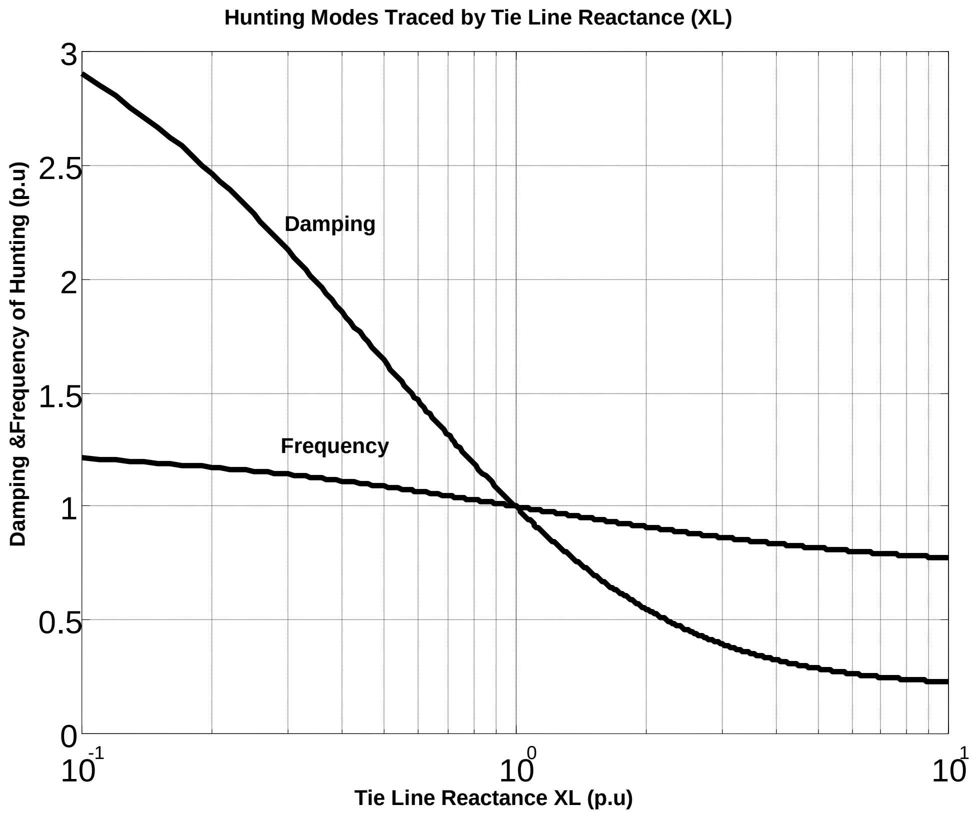

Hunting Modes Traced by Q-Damper Resistance (Rkq)
Damping & Frequency of hunting (p.u.)
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
10-1
100
Q-Damper Resistance (p.u.)

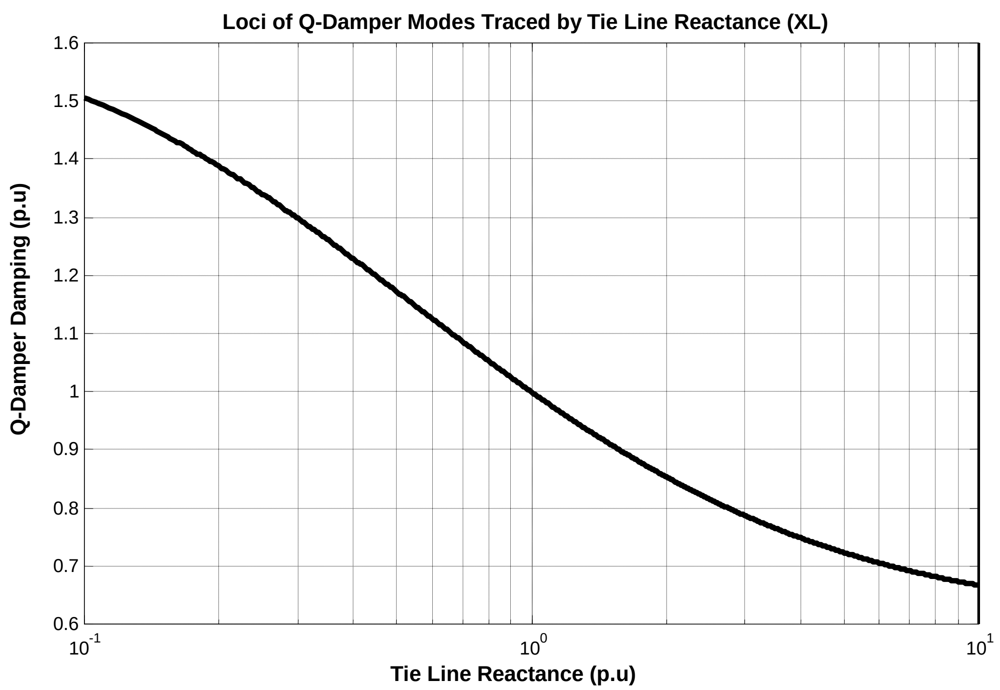

Hunting Modes Traced by Field Resistance (RF)
Damping & Frequency Of Hunting (p.u.)
1.25
1.2
1.15
1.1
1.05
1
0.95
10^-1
10^0
10^1
Field Resistance (p.u.)

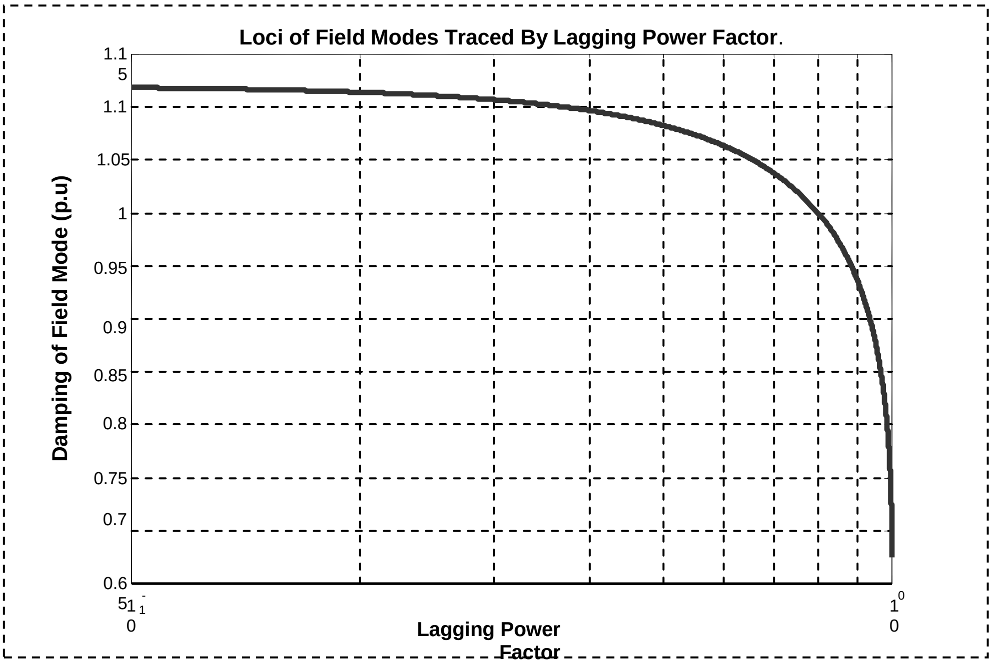

Loci of Field Modes Traced by Field Resistance (RF)
Damping of Field Mode (p.u)
7
6
5
4
3
2
1
0
10^-1
10^0
10^1
Field Resistance (p.u)

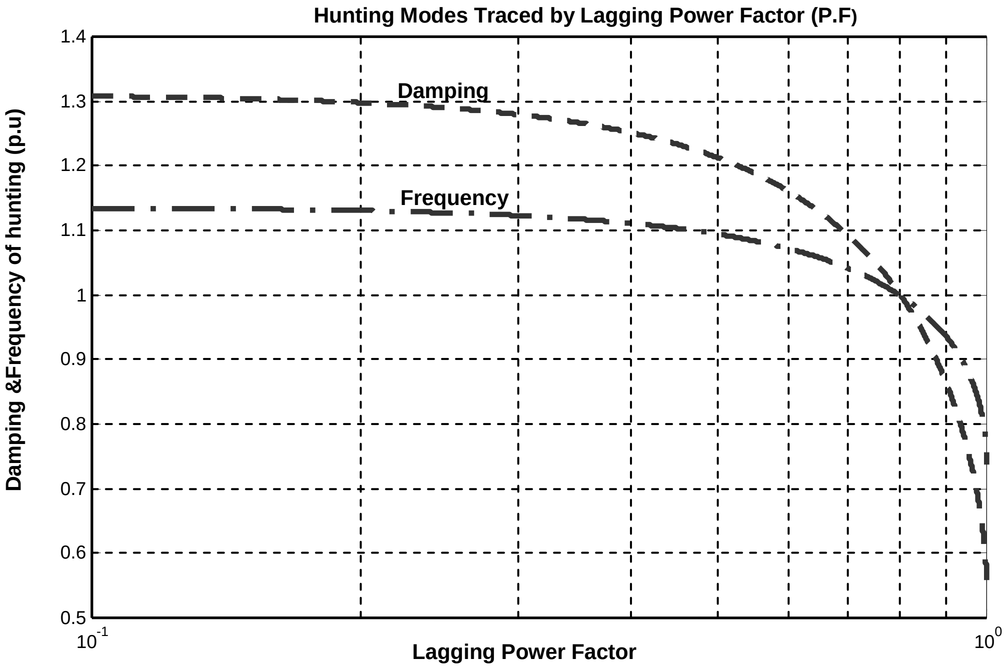

Stator Modes Traced by Tie Line Reactance (XL)
Damping frequency of Stator Modes (p.u)
Damping
Frequency
Tie Line Reactance XL (p.u)


Loci of Field Modes Traced by Tie Line Reactance (XL)
Damping of Field Mode (p.u)
Tie Line Reactance (p.u)


Hunting Modes Traced by Tie Line Reactance (XL)
Damping
Frequency
Damping &Frequency of Hunting (p.u)
Tie Line Reactance XL (p.u)


Loci of Q-Damper Modes Traced by Tie Line Reactance (XL)
Q-Damper Damping (p.u)
Tie Line Reactance (p.u)





1. **O.Saito, H.Mukae, K.Murotani**," Suppression of Self Excited Oscillations in Series Compensated Transmission Lines By Excitation Control of Synchronous Machine", *IEEE Transactions on Power Systems*, Vol.PAS-94, N°5, September/October 1975, pp.1777-1788.